Cheltenham Girls High School

Student Name: ....

Class: ....

Teacher ....

Mathematics Extension 1

Trial Examination 2010

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total marks – 84

- Attempt Questions 1–7
- All questions are of equal value

Question	1	2	3	4	5	6	7	Total	%
Marks	/12	/12	/12	/12	/12	/12	/12	/84	

Question 1: (12 Marks) **Start a new Answer Sheet**

(a) Differentiate $y = e^x sin^{-1}x$ **2**

(b) Using the substitution $u = 4 - x^2$, evaluate the following: **3**

$$\int_0^2 \frac{xdx}{\sqrt{4 - x^2}}$$

(c) For the function $y = 2cos^{-1}4x$

i. State the domain and range. **2**

ii. Draw a neat sketch of $y = 2cos^{-1}4x$ **2**

(d) Simon is preparing to study for his trial examinations. He has borrowed 3 Mathematics, 5 English and 2 Science books from the library. Each of the books is different.

i. How many ways can he arrange the books on his shelf if there are no restrictions? **1**

ii. How many ways are there of arranging the books on the shelf if a Science books is at each end and the Maths and English books are kept in subject areas. **2**

Question 2: (12 Marks) **Start a new Answer Sheet**

(a) The acute angle between the tangents to the curves $y = e^{2x}$ and $y = e^{-mx}$ at $x = 0$ is 45°.

Find the values of m. **4**

(b) The equation $2x^3 + 6x^2 + 9x - 2 = 0$ has roots α, β and γ. Find:

i. $\alpha + \beta + \gamma$ **1**

ii. $\alpha\beta + \beta\gamma + \gamma\alpha$ **1**

iii. $\alpha^2 + \beta^2 + \gamma^2$ **2**

(c) Find the exact value of: $\cos 75°$ **2**

(d) Given that $tanA$ and $tanB$ are the roots of the equation $2x^2 - 3x - 1 = 0$; **2**

Find the value of $tan(A + B)$

Question 3: (12 Marks) **Start a new Answer Sheet**

(a) Given the function $y = \sin^{-1}\frac{x}{2}$; find:

i. $\frac{dy}{dx}$. **1**

ii. The gradient at $x = \sqrt{3}$. **2**

iii. The equation of the normal in exact form. **3**

(b) i. Differentiate $y = -4x + 4x\log_e 4x$ **2**

ii. Using the result in (i) find the minimum value for $y = 4x\log_e 4x - 4x$ **2**

(c) Evaluate $\lim_{x\to 0}\left(\frac{\sin 3x}{2x}\right)$ **2**

Question 4: (12 Marks) **Start a new Answer Sheet**

(a) Two points on the parabola $x^2 = 4ay$ are P and Q and have coordinates $(2ap, ap^2)$ and $(2aq, aq^2)$ respectively.

i. Find the equation of the chord PQ. **2**

ii. Find the coordinates of the midpoint $M(x, y)$ of PQ. **1**

iii. If $\angle POQ = 90^\circ$, where O is the origin, prove that $pq = -4$. **1**

iv. Hence find the locus of M as P and Q move along the parabola. **2**

(b) Use Newton's method to find a second approximation to the positive root of $sinx + x - 2 = 0$. Take $x = 1.1$ as the first approximation. **2**

(c) Prove by mathematical induction that: **4**

$$\sum_{r=1}^{n}(3r-1) = \frac{3n^2+n}{2} \qquad n \geq 1$$

Question 5: (12 Marks) **Start a new Answer Sheet**

(a) A cross section of a termites mound is found to have equation $y = \dfrac{15}{8+2x^2}$.

i. Draw a neat sketch of $y = \dfrac{15}{8+2x^2}$ between $x = -2\sqrt{3}$ and $x = 2\sqrt{3}$. **2**

ii. Calculate the area of the cross-section between $x = -2\sqrt{3}$ and $x = 2\sqrt{3}$. **3**

(b) Solve the equation $\sqrt{3}sinA + cosA = 1$ in the domain $0 \leq A \leq 2\pi$ **4**

(c) Using the substitution $x = 3\cos\theta$ show that:

$$\int \frac{x^2 dx}{\sqrt{9-x^2}} = -\frac{9}{2}\cos^{-1}\frac{x}{3} - \frac{1}{2}x\sqrt{9-x^2} + c$$ **3**

Question 6: (12 Marks) **Start a new Answer Sheet**

(a) For the function $f(x) = \frac{x^2}{x^2-4}$

i. Show the function is an even function. **1**

ii. State any vertical asymptotes. **1**

iii. Find the horizontal asymptotes. **1**

iv. Find where the graph cuts the y axis. **1**

v. Draw a neat sketch of the function $f(x) = \frac{x^2}{x^2-4}$ **2**

(b) Evaluate $\int_0^{\frac{\pi}{3}} sin^2 x\, dx$ **2**

(c) The perpendicular height of a jet above the ground is 2000 metres. An observer due east of the jet looks up at the jet at an angle of elevation of 60°. A second person looks up at the jet at an angle of elevation of 30°. If the two people subtend an angle of 80° at the base of the perpendicular below the jet, calculate:

i. The exact length of BD and BC. **2**

ii. The distance between the two people, to the nearest metre. **2**

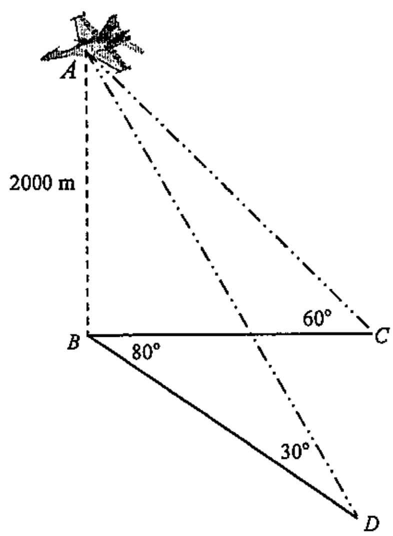
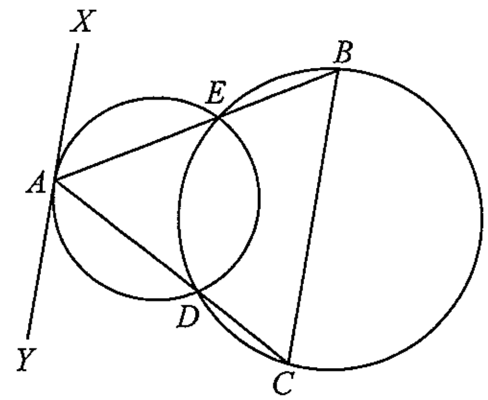


Diagram not to scale.

Question 7: (12 Marks) **Start a new Answer Sheet**

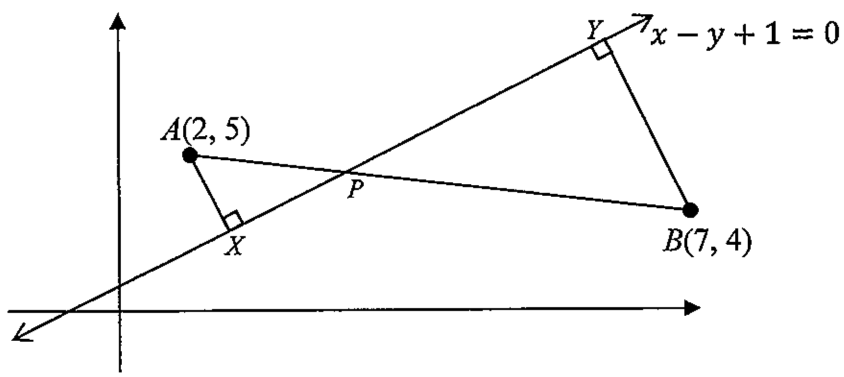
(a) Two circles intersect at E and D. From a point A two lines are drawn through the points of intersection to meet the other circle at B and C as shown in the diagram.

Prove the tangent XY at A is parallel to the line BC. **3**



(b) Solve the inequation $\left|2x-\frac{1}{2}\right|>\sqrt{x-x^2}$ **4**

(c) The points $A(2, 5)$ and $B(7, 4)$ are on either side of the line; $x-y+1=0$.



i. Prove $\Delta AXP \parallel\!\mid \Delta BYP$ **2**

ii. **Hence** find the coordinates of P that divides the interval AB. **3**

End of Examination

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}}\,dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}}\,dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

2, 3, 5, 6

Solutions Trial HSC Mathematics Extension 1

Question 1

(a) $\frac{d}{dx}(e^x \sin^{-1}x) = (\sin^{-1}x)(e^x) + (e^x)\left(\frac{1}{\sqrt{1-x^2}}\right)$ **2 Marks – correct answer**

$\frac{d}{dx}(e^x \sin^{-1}x) = e^x\left[(\sin^{-1}x) + \left(\frac{1}{\sqrt{1-x^2}}\right)\right]$ **1 Mark – correctly finding** *u′ and v′*

(b) $u = 4 - x^2$

$\frac{du}{dx} = -2x$

$-\frac{du}{2} = xdx$

1 Mark

$x = 2 \quad u = 0$

$x = 0 \quad u = 4$

$$\int_0^2 \frac{xdx}{\sqrt{4-x^2}} = -\frac{1}{2}\int_4^0 \frac{du}{\sqrt{u}}$$

$$\int_0^2 \frac{xdx}{\sqrt{4-x^2}} = \frac{1}{2}\int_0^4 u^{-\frac{1}{2}}du$$

$$\frac{1}{2}\int_0^4 u^{-\frac{1}{2}}du = \frac{1}{2}\left[2\sqrt{u}\right]_0^4$$ **1 Mark**

$$\frac{1}{2}\left[2\sqrt{u}\right]_0^4 = \frac{1}{2}[(4) - (0)]$$

$$\therefore \int_0^2 \frac{xdx}{\sqrt{4-x^2}} = 2$$ **1 Mark**

(c) $y = 2\cos^{-1}4x$

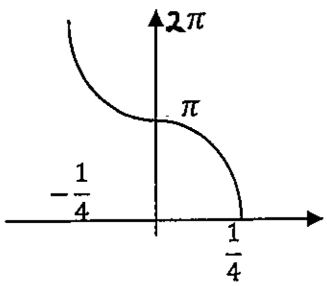
i. $D: -1 \le 4x \le 1$

1 Mark

$D: -\frac{1}{4} \le x \le \frac{1}{4}$

$R: 2(0) \le y \le 2(\pi)$ **1 Mark**

$R: 0 \le y \le 2\pi$



1 Mark – correct graph

1 Mark – labelling axes

(d) i. No restrictions = 10! **1 Mark**

ii. N^o ways = $(2!)[(2!)(3!)(5!)]$? **2 Marks – correct working and answer**

1 Mark – some correct working with explanation

Question 2

(a)

$y = e^{2x}$

$\frac{dy}{dx} = 2e^{2x}$ at $x = 0$

$m_1 = 2$

$y = e^{-mx}$

$\frac{dy}{dx} = -me^{-mx}$ at $x = 0$ **1 Mark**

$m_2 = -m$

$tan\alpha = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$tan45 = \left|\frac{2+m}{1-2m}\right|$

$1 = \left|\frac{2+m}{1-2m}\right|$ **1 Mark**

$\frac{2+m}{1-2m} = 1$ $\frac{2+m}{1-2m} = -1$

$2 + m = 1 - 2m$ $2 + m = 2m - 1$ **1 Mark**

$3m = -1$ $3 = m$

$m = \frac{-1}{3}$ $m = 3$

Since the gradient must be negative $m = -\frac{1}{3}$ **1 Mark**

(b) $2x^3 + 6x^2 + 9x - 2 = 0$

i. $\alpha + \beta + \gamma = \frac{-6}{2} = -3$ **1 Mark**

ii. $\alpha\beta + \beta\gamma + \alpha\gamma = \frac{9}{2}$ **1 Mark**

iii. $\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \alpha\gamma)$

$= (-3)^2 - 2\left(\frac{9}{2}\right)$ **1 Mark**

$= 0$ **1 Mark**

(c) $cos(30+45) = cos30cos45 - sin30sin45$ **1 Mark**

$cos(30+45) = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$

$cos(30+45) = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$cos(30+45) = \frac{\sqrt{6}-\sqrt{2}}{4}$ **1 Mark**

(d) $2x^2 - 3x - 1$

$\alpha = tanA$ and $\beta = tanB$

$\alpha + \beta = tanA + tanB = \frac{3}{2}$

$\alpha\beta = tanAtanB = -\frac{1}{2}$

$\tan(A+B) = \frac{tanA+tanB}{1-tanAtanB}$

$\tan(A+B) = \frac{\frac{3}{2}}{1-\left(-\frac{1}{2}\right)}$ **1 Mark**

$\tan(A+B) = \frac{\left(\frac{3}{2}\right)}{\left(\frac{3}{2}\right)}$

$\tan(A+B) = 1$ **1 Mark**

Question 3

(a)

i. $\frac{dy}{dx} = \frac{1}{\sqrt{4-x^2}}$ **1 Mark**

ii. $m = \frac{1}{\sqrt{4-(\sqrt{3})^2}}$ **1 Mark**

$m = 1$ (tangent) **1 Mark**

$m = -1$ (normal)

iii. At $x = \sqrt{3}, y = sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$ **1 Mark**

$y - \frac{\pi}{3} = -1(x - \sqrt{3})$ **1 Mark**

$3y - \pi = -3x + 3\sqrt{3}$

$3x + 3y - 3\sqrt{3} - \pi = 0$ **1 Mark**

(b) $\frac{dy}{dx} = \left[(log_e 4x)(4) + (4x)\left(\frac{4}{4x}\right)\right] - 4$ **1 Mark**

$\frac{dy}{dx} = 4log_e 4x$ **1 Mark**

(c) Min value will occur at the turning point

$\frac{dy}{dx} = 4log_e 4x$

$4log_e 4x = 0$

$log_e 4x = 0$

$e^{log_e 4x} = e^0$

$4x = 1$

$x = \frac{1}{4}$ **1 Mark**

When $x = \frac{1}{4}, y = 4\left(\frac{1}{4}\right) log_e\left(4.\frac{1}{4}\right) - 4\left(\frac{1}{4}\right) = -1$

Therefore the minimum value is $y = -1$ **1 Mark**

(d)

$\lim_{x\to 0}\left(\frac{sin3x}{2x}\right) = \left(\frac{3}{2}\right)\lim_{x\to 0}\left(\frac{sin3x}{3x}\right)$ **1 Mark**

$= \frac{3}{2}(1)$

$= \frac{3}{2}$ **1 Mark**

Question 4

(a)

i. $\frac{y-ap^2}{x-2ap} = \frac{aq^2-ap^2}{2aq-2ap}$ **1 Mark**

$\frac{y-ap^2}{x-2ap} = \frac{a(q-p)(q+p)}{2a(q-p)}$

$y - ap^2 = \frac{(q+p)}{2}[x - 2ap]$

$y - ap^2 = \frac{(q+p)x}{2} - 2apq - ap^2$

$y = \frac{(q+p)x}{2} - \cancel{2}apq$ **1 Mark**

ii. Midpoint $= \left(\frac{2ap+2aq}{2}, \frac{ap^2+aq^2}{2}\right)$

Midpoint $= \left(a(p+q), \frac{ap^2+aq^2}{2}\right)$ **1 Mark**

iii. $m_1 = \frac{ap^2-0}{2ap-0} = \frac{p}{2}$ and likewise $m_2 = \frac{q}{2}$

$m_1 \times m_2 = -1$

$\frac{p}{2} \times \frac{q}{2} = -1$

$pq = -4$

1 Mark

iv. $x = a(p+q)$

$\frac{x}{a} = (p+q)$

$\left(\frac{x}{a}\right)^2 = (p+q)^2 \quad ...(1)$

$y = \frac{a}{2}(p^2+q^2) \quad ...(2)$

$y = \frac{a}{2}[(p+q)^2 - 2pq]$

$y = \frac{a}{2}\left[\left(\frac{x}{a}\right)^2 - 2(-4)\right]$

1 Mark – correctly sub (1) into (2)

$y = \frac{x^2}{2a} + 4a$

$2ay = x^2 + 8a^2$

$2ay - 8a^2 = x^2$

Therefore the locus is:

$x^2 = 2a(y - 4a)$

1 Mark - answer

(b)

$x_2 = (1.1) - \frac{\sin(1.1)+1.1-2}{+\cos(1.1)+1}$

1 Mark

$x_2 = (1.1) - \frac{(-0.00879\ ...)}{~~(0.5464\ ...)~~}$ 1.4536

$x_2 =$ ~~(1.1) + 0.01608~~...

$x_2 =$ ~~1.11608~~... 1.106

$x_2 =$ ~~1.11~~

1 Mark

(c) $2 + 5 + \cdots + 3n - 1 = \frac{3n^2+n}{2}$

We need to prove that $S_{n+1} = S_n + T_{n+1}$

Step 1: Show true for $n = 1$

$LHS = 2$ $\qquad$ $RHS = \frac{3+1}{2} = 2$ **1 Mark**

Step 2: Assume true for $n = k$

(a) $2 + 5 + \cdots + 3k - 1 = \frac{3k^2+k}{2}$ **1 Mark**

Step 2: Prove true for $n = k + 1$

$S_{k+1} = \frac{3(k+1)^2+(k+1)}{2}$ $\qquad$ $S_n + T_{n+1} = \frac{3k^2+k}{2} + 3(k+1) - 1$

$$S_n + T_{n+1} = \frac{3k^2+k+6k+6-2}{2}$$

2marks ~~**1 Mark**~~

$$S_n + T_{n+1} = \frac{3k^2+7k+4}{2}$$

$$S_n + T_{n+1} = \frac{3k^2+6k+3+k+1}{2}$$

$$S_n + T_{n+1} = \frac{(3k^2+6k+3)+(k+1)}{2}$$

$$S_n + T_{n+1} = \frac{(3(k+1)^2)+(k+1)}{2}$$

$\therefore S_{n+1} = S_n + T_{n+1}$

Step 4: Conclusion

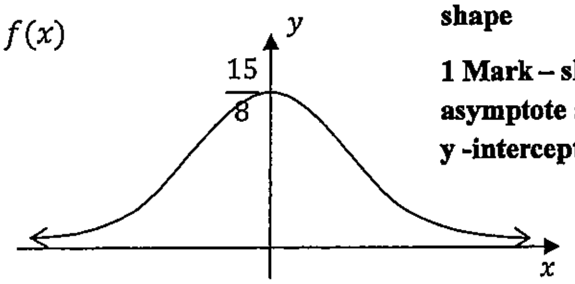
Hence if the statement is true for n = k, then it is also true when $n = k + 1$. The statement is true for $n = 1$ and so it is true for $n = 2$ and so on. Hence it is true for all n. ~~**1 Mark**~~

Question 5:

i. The function is even as $f(-x) = f(x)$

$\lim_{x\to\infty}\left(\frac{\frac{15}{x^2}}{\frac{8}{x^2}+2}\right) = \frac{0}{2} = 0$

$x = 0, y = \frac{15}{8}$



1 Mark – correct shape

1 Mark – showing asymptote and y -intercept

ii.

$A = 2\int_0^{2\sqrt{3}} \frac{15}{2(4+x^2)}dx$ **1 Mark**

$A = 15\int_0^{2\sqrt{3}} \frac{1}{4+x^2}dx$

$A = 15\left[\frac{1}{2}tan^{-1}\left(\frac{x}{2}\right)\right]_0^{2\sqrt{3}}$ **1 Mark**

$A = 15\left[\left(\frac{1}{2}\right)tan^{-1}(\sqrt{3}) - \left(\frac{1}{2}\right)tan^{-1}(0)\right]$

$A = 15\left[\left(\frac{1}{2}\right)\left(\frac{\pi}{3}\right) - \left(\frac{1}{2}\right)(0)\right]$

1 Mark

$\therefore A = \frac{15\pi}{6}$ units^2 or $\frac{5\pi}{2}$ or $7\cdot85$

(b) $R = \sqrt{(\sqrt{3})^2 + 1^2} = 2$ **1 Mark**

$tan\alpha = \frac{1}{\sqrt{3}} = \frac{\pi}{6}$

$\sqrt{3}sinA + cosA \equiv 2sin\left(A+\frac{\pi}{6}\right) = 1$

$sin\left(A+\frac{\pi}{6}\right) = \frac{1}{2}$ **1 Mark**

<u>**1st Quad**</u>.

$\left(A+\frac{\pi}{6}\right) = sin^{-1}\frac{1}{2}$

$\left(A+\frac{\pi}{6}\right) = \frac{\pi}{6}$

1 Mark - A_1 *and* A_2

$A_1 - \frac{\pi}{6} = \frac{\pi}{6}$ $\left(A_2+\frac{\pi}{6}\right) = \pi - \frac{\pi}{6}$ <u>Check $x = 2\pi$</u>

$A_1 = 0$ $A_2 = \frac{2\pi}{3}$ $\sqrt{3}sin2\pi + cos\pi = 0 + 1 = 1$ ✓

Therefore the solutions are: $A = 0,\ \frac{2\pi}{3}\ and\ 2\pi$ **1 Mark** $= 2\pi$

(c)

$x = 3cos\theta$

$\frac{dx}{d\theta} = -3sin\theta$

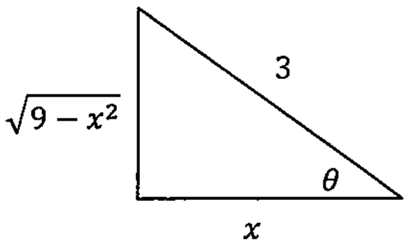
$dx = -3sin\theta d\theta$

1 Mark – finding needed expressions

$x^2 = 9cos^2\theta$

$x = 3cos\theta$

$\theta = cos^{-1}\left(\frac{x}{3}\right)$



$\int \frac{x^2}{\sqrt{9-x^2}}dx = \int \frac{9cos^2\theta}{\sqrt{9-9cos^2\theta}}(-2sin\theta)d\theta$

$= \int \frac{9cos^2\theta}{\sqrt{9sin^2\theta}}(-3sin\theta)d\theta$

$= \int \frac{9cos^2\theta}{3sin\theta}(-3sin\theta)d\theta$

$= \int -9cos^2\theta\, d\theta$

$= -9\left[\frac{\theta}{2} + \frac{1}{4}sin2\theta\right] + c$

1 Mark – correct integration

$= -9\left[\frac{\theta}{2} + \left(\frac{1}{4}\right)(2)sin\theta cos\theta\right] + c$

$= -9\left[\frac{1}{2}cos^{-1}\left(\frac{x}{3}\right) + \frac{1}{2}\left(\frac{\sqrt{9-x^2}}{3}\right)\left(\frac{x}{3}\right)\right] + c$

$= -\frac{9}{2}cos^{-1}\left(\frac{x}{3}\right) - \frac{x}{2}\left(\frac{\sqrt{9-x^2}}{3}\right) + c$

1 Mark – correct answer

Question 6

(a)

i. $f(-x) = \frac{(-x)^2}{(-x)^2-4}$

$f(-x) = \frac{x^2}{x^2-4}$ **1 Mark**

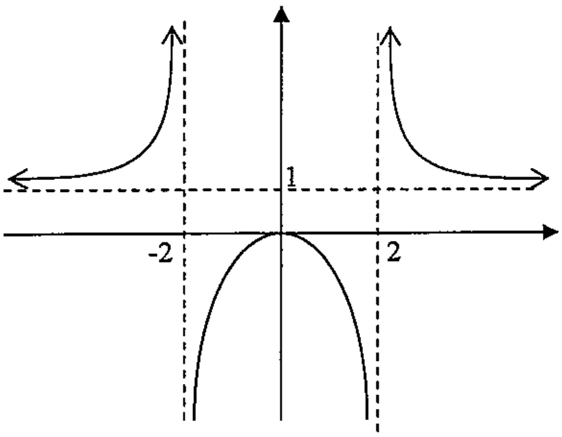
$f(-x) = f(x)$

ii. $x \neq \pm 2$ **1 Mark**

iii. $\lim_{x\to\infty}\left(\frac{\frac{x^2}{x^2}}{\frac{x^2}{x^2}-\frac{4}{x^2}}\right) = \frac{1}{1-0} = 1$ **1 Mark**

iv. $x = 0, \ y = 0$ **1 Mark**

v.



1 Mark – correct graph

1 Mark – labelling

(c)

i. $tan60 = \frac{2000}{BC}$ $\qquad tan30 = \frac{2000}{BD}$

$BC = \frac{2000}{tan60}$ $\qquad BD = 2000\sqrt{3}$

$BC = \frac{2000}{\sqrt{3}}$

1 Mark - BC

1 Mark - BD

ii. $(DC)^2 = \left(\frac{2000}{\sqrt{3}}\right)^2 + \left(2000\sqrt{3}\right)^2 - 2\left(\frac{2000}{\sqrt{3}}\right)\left(2000\sqrt{3}\right)cos80$ **1 Mark - BC**

iii. $(DC)^2 =$ 11944·1479...

$DC =$ 3456 m **1 Mark - BC**

(b)

$$\int_0^{\frac{\pi}{3}} sin^2x dx = \left[\frac{x}{2} - \frac{1}{4}sin2x\right]_0^{\frac{\pi}{3}}$$

$$\left[\frac{x}{2} - \frac{1}{4}sin2x\right]_0^{\frac{\pi}{3}} = \left[\left(\frac{\pi}{6} - \frac{1}{4}\left(\frac{\sqrt{3}}{2}\right)\right)\right]$$

$$\int_0^{\frac{\pi}{3}} sin^2x dx = \left(\frac{4\pi - 3\sqrt{3}}{24}\right)$$

Question 7

(a)

$\angle YAD = \angle AED = x$ (angle b/w a tangent and chord equals the angle in the alt. segment) **1 M**

$\angle BED = 180 - x$ (straight line = 180)

$\angle BCD = x$ (EBCD is a cyclic qual, opposite angle of cyclic quad are supplementary.) **1 M**

$\therefore \angle YAD = \angle BCD = x$

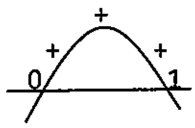
$\therefore XY \parallel BC$ (Alternate angles) **1 Mark**

(b) Domain of $\sqrt{x - x^2}$

$x - x^2 \geq 0$

$x(1 - x) \geq 0$

D: $0 \leq x \leq 1$



1 Mark

$$\left|2x - \frac{1}{2}\right| = \sqrt{x - x^2}$$

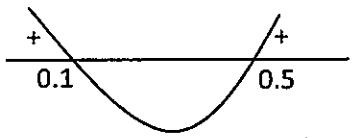
$$\left(2x - \frac{1}{2}\right)^2 = \left(\sqrt{x - x^2}\right)^2$$

$$4x^2 - 2x + \frac{1}{4} = x - x^2$$

1 Mark

$20x^2 - 12x + 1 > 0$

$(10x - 1)(2x - 1) > 0$



$x < 0.1 \;\; and \;\; x > 0.5$

1 Mark

Putting both answers together we get:

$0 \leq x < \frac{1}{10}$ and $\frac{1}{2} < x \leq 1$ **1 Mark**

(c)

$\angle AXP = \angle BYP = 90°$ (given) **1 Mark**

$\angle APX = \angle BPY$ (Vert. opp angles are equal) **1 Mark**

$\therefore \Delta XAP \,|||\, \Delta BYP$

$d_{p_{(AX)}} = \frac{|2-5+1|}{\sqrt{2}}$

$d_{p_{(AX)}} = \frac{2}{\sqrt{2}} = \sqrt{2}$ likewise $d_{p_{(YB)}} = 2\sqrt{2}$ **1 Mark**

Therefore the ratio of $AP:PB$ is 1:2 (sides of similar triangles are in same ratio) **1 Mark**

$P\left(\frac{1(7)+2(2)}{3}, \frac{1(4)+2(5)}{3}\right)$

1 Mark

$\therefore P\left(\frac{11}{3}, \frac{14}{3}\right)$

Q3

a) (i) $\dfrac{1}{\sqrt{1-\frac{x^2}{4}}} \times \frac{1}{2}$ 1

$y' = \dfrac{1}{\sqrt{4-x^2}}$

(ii) $y' = 1$ 2

(iii) $m_2 = -1$

$y - \frac{\pi}{3} = -1(x - \sqrt{3})$ 3

$y = -x + \sqrt{3} + \frac{\pi}{3}$ or

$3x + 3y - 3\sqrt{3} - \pi$

b)

$u = 4x \quad u' = 4$

$v = \ln 4x \quad v' = \frac{1}{x}$

$y' = 4\ln 4x + 4 - 4$ 2

$y' = 4\ln 4x = 0$

(ii) $x = \frac{1}{4}$ $y = -1$ ✓

$y'' = \frac{4}{x}$ 2

$y''(\frac{1}{4}) = 16 > 0$

$\therefore$ Min Val at $(\frac{1}{4}, -1)$

(c)

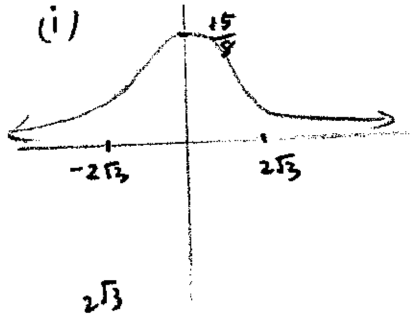
$= \lim_{x \to 0} \dfrac{\sin 3x}{3x} \times \dfrac{3}{2}$

$= \frac{3}{2}$ 2

Q5

a) $y = \dfrac{15}{8 + 2x^2}$

(i)



2

$A = 2\int_0^{2\sqrt{3}} \frac{15}{2} \cdot \frac{dx}{4 + x^2}$

$= 15 \left[\frac{1}{2}\tan^{-1}\frac{x}{2}\right]_0^{2\sqrt{3}}$

$= \frac{15}{2}\left(\frac{\pi}{3}\right) = \frac{5\pi}{2}$ 3

$\doteqdot 7.85$

(b) $R = \sqrt{4} = 2$

$\tan\alpha = \frac{1}{\sqrt{3}} = \frac{\pi}{6}$

$2\sin(A + \frac{\pi}{6}) = 1$

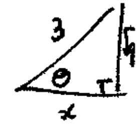
$A + \frac{\pi}{6} = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}$ 4

$A = 0, \frac{4\pi}{6}, 2\pi$

$3\frac{1}{2}$

$\cos^{-1}\frac{x}{3} = \theta$

$\frac{x}{3} = \cos\theta$



2

(c) $\frac{dx}{d\theta} = -3\sin\theta$

$= \int \dfrac{9\cos^2\theta \cdot -3\sin\theta \, d\theta}{\sqrt{9 - 9\cos^2\theta}}$ 1

$= \int \dfrac{-27\cos^2\theta\sin\theta}{3\sin\theta} \cdot d\theta$

$= -9\int \cos^2\theta \, d\theta$

$= \frac{-9}{2}\int 1 + \cos 2\theta \, d\theta$

$= \frac{-9}{2}\left(\theta + \frac{1}{2}\sin 2\theta\right) + C$

$= \frac{-9}{2}\left(\cos^{-1}\frac{x}{3} + \frac{x}{3} \cdot \frac{\sqrt{9-x^2}}{3}\right) + C$

(3) $= -\frac{9}{2}\cos^{-1}\frac{x}{3} - \frac{1}{2}x\sqrt{9-x^2} + C$